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PRELIMINARY TEST ON THE STEERING ALGORITHM FOR KEEPING A TIME SCALE SYNCHRONIZED TO UTC

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***Abstract* – This paper deals with some test which are currently under way at the IEN in Italy with the aim of identifying a suitable algorithm for steering the local UTC(IEN) to the UTC. The tests are performed with the IEN experimental data of commercial Cesium clock for a period of about 2 years and of H-Maser clock for a period of about 3 months.**
***Keywords* –Extrapolation, Allan variance, Wiener Process.**

I. INTRODUCTION

In recent days the interest in keeping a local time scale strictly synchronized to the international UTC is becoming stronger due to some new demanding applications as the European GNSS project Galileo and also the international Mutual Recognition Arrangement asking for “equivalent” standards.

Some test are currently under way at the IEN in Italy with the aim of identifying a suitable algorithm for steering the local UTC(IEN) to the UTC.
Since the international UTC is known only a posteriori while the local time scales are generally realized in real time, a prediction of the UTC and local clock behavior for a period of about 45 days is needed. When, a posteriori, BIPM data are available the difference of the prediction versus the true behavior of UTC-UTC(IEN) is evaluated. Such difference “prediction-reality” is then statistically estimated.

The current tests follow two approaches:

1. From the past BIPM data giving UTC-UTC(IEN) an extrapolation for the subsequent days is evaluated
2. The behavior of the clocks is modeled by a stochastic process where also some deterministic parameters as frequency offset and drift are introduced. The study of this clock model allows also the evaluation of the best clock prediction and of its uncertainty.

The tests are performed with IEN experimental data of commercial Cesium clock for a period of about 2 years and of a newly acquired H-Maser clock for a period of about 3 months.
In the paper we explain the mathematical methods used for these tests and the results obtained on experimental IEN data. Firstly we use the linear extrapolation for the prediction of the clock behavior. Secondly, the clock behavior is modeled by a stochastic process. The study of this clock model allows the evaluation of the clock prediction and of its uncertainty.

We can anticipate that we found that for the Cesium clocks, their prediction with respect to UTC for a period of 45 days has an error with standard deviation of about 30 ns. This means that a prediction error with 95% confidence level would be around 60 ns. For the newly acquired H-Maser clock, 3 months of data are not sufficient to assess a significant statistical analysis. The refinement of the results are possible with a larger set of data and their evaluation is in progress.

II. FIRST METHOD: LINEAR EXTRAPOLATION

Two different mathematical methods are introduced in order to evaluate the feasibility of an UTC prediction and a consequent frequency steering of a Cesium or a H-Maser.
A first test consists in using past UTC data to predict the future by a simple extrapolation.

The test was performed by using past BIPM data giving UTC-UTC(IEN) spaced by 5 days on BIPM standard dates (MJD ending by 4 and 9). From those past data a prediction of the difference UTC-UTC(IEN) was evaluated for the subsequent 45 day period. When, a posteriori, BIPM data are available, the difference of the prediction versus the true behavior of the UTC-UTC(IEN) is evaluated. Such difference “prediction-reality” is statistically estimated.
As a first attempt, the linear extrapolation is evaluated on one month of past data.
Using the 6 BIPM past data corresponding to 1 month of measurements of UTC-UTC(IEN), a prediction has been evaluated for the following 45 days (9 dates spaced by 5 days). The test was carried out on around 1.5 years of data from 5/01/2001 (MJD: 51914) to 30/05/2002 (MJD: 52424). The root mean square of the difference between the true and the predicted values (9 differences in the 45 day period) was estimated for any 45 day period. In the end, the mean of the root mean square error on the entire 1.5 year period was estimated. In the Fig. 1 the differences UTC-UTC (IEN) with their predicted values are shown. The real clock data (blue color) are reported together with the 45 days prediction values (colored lines). We can see that the linear extrapolation of the cesium behavior in some cases fits reasonably well the true clock behavior, while in other cases, that are reasonably possible due to the cesium random behavior, the linear extrapolation is not able to track random variations.

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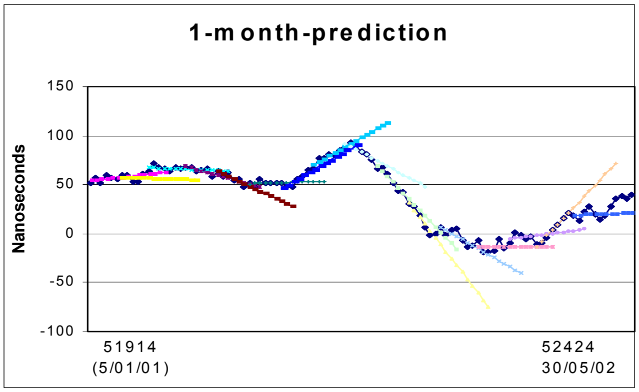

Figure 1. Predicted and real values of UTC-UTC(IEN) (estimation period 1 months).

As a second step, the linear extrapolation of past BIPM data was estimated on the previous 3 months of data (instead than one month) to check if a longer past period was helping in improving the prediction. Using the 18 data spaced by 5 days included in the previous 3 months period of BIPM data, a prediction has been made for the following 45 days of data (9 dates spaced by 5 days). The root mean square of the difference between the true values and that predicted was estimated. Also the mean of all these prediction errors over the entire 1.5 years period was estimated.

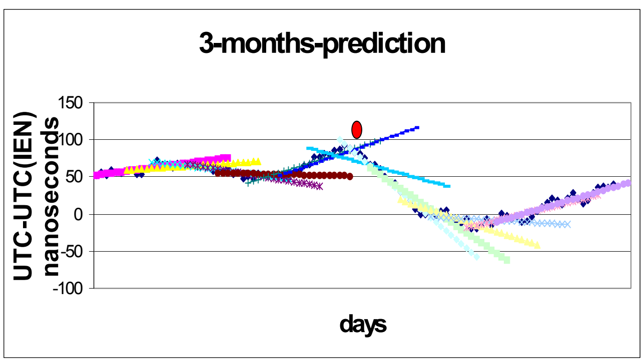

Figure 2. Predicted and real values of UTC-UTC(IEN) (estimation period 3 months).

The mean of the root mean squares of prediction errors on the whole 1.5 years is 20.03 ns. Even worse that before. We can see that using 3 months of past BIPM data does not improve the quality of the prediction.

The IEN H-Maser is newly installed. We have only 3 month of data available. Therefore the results obtained by these tests are not statistically significant but they represents a first step in this project. To have a larger number of experimental data, we used the measures of the H maser versus the UTC(IEN). The IEN H-Maser is measured twice a day, every 12 hours, versus UTC(IEN). The IEN data cover a period of 3 months but with sampling rate of 0.5 days. Data are available since 26/10/2002 (MJD: 52573) to 31/01/2003 (MJD: 52669).

The H-maser frequency measures show a frequency drift corrupted by white FM noise.

As a first attempt, the linear frequency drift is evaluated on one month of past daily frequency data obtained from the measures UTC(IEN)-H-maser. From those past data a prediction of the difference UTC(IEN)-H-Maser was evaluated for the subsequent 30 day period. This process is repeated for the whole observation period of 3 months. The difference of the frequency predicted values versus the true behavior was then statistically estimated. The prediction was firstly evaluated on frequency data then, by integration, also a time prediction was generated.

In the Fig. 3 the frequency measures and the predictions are shown. The drift of the maser on 30/01/2003 (MJD: 52669) was estimated to be $d=5.23*10^{-15}$/day.

The root mean square of the difference between the true frequency values and those predicted was estimated for any months and also the mean of those root mean squares on the entire 3 months period. The mean of the root mean square of the differences (prediction-reality) of the frequency values is estimated to be $2.34*10^{-14}$.

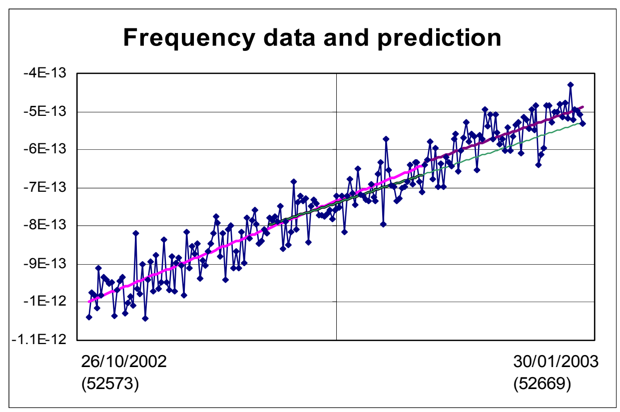

Figure 3. Predicted and real values of the H maser frequency values estimated versus UTC(IEN)

After the analysis in the frequency data we have analyzed the time data. Integrating the estimate of the frequency values we obtained the estimate on the time difference UTC(IEN) – Hmaser (Fig. 4). Every month the frequency and time predictions are updated considering the new month of data. In this case, the mean value of the time prediction error of the whole 3 months period is 20 nanoseconds and the standard deviation is 32 nanoseconds. These prediction errors are not statistically significant because 3 months of data are a very small ensemble. For completeness we evaluated also a frequency prediction on the measures (UTC–Hmaser) but the observation period was at the moment too short for inferring a significant statistical behavior.

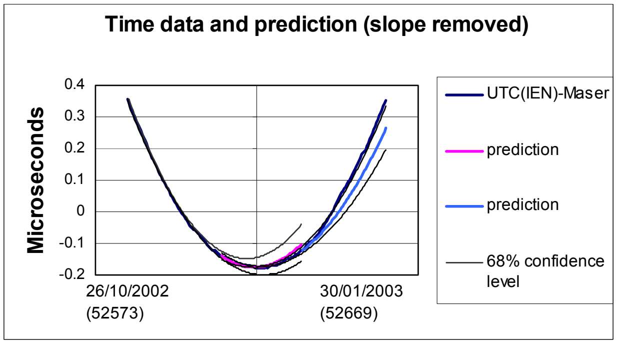

Figure 4. Predicted and real values of the differences UTC(IEN)-H maser.

III. SECOND METHOD: CLOCK MODEL

By a more scientific analysis of these data, it comes out that the prediction error is of the order of random behavior of the clocks and therefore with one single clock this is the best possible prediction error and longer past observation period are not helping. We studied that in details for the cesium clocks.

To this aim the random and the deterministic behavior of Cesium clocks are evaluated. Secondly, after having identified the best clock model, the best prediction is also examined and quantified.

To have more experimental data at disposal, we used the measures (UTC(IEN)- Cs_i) taken every 12 hours for the some 1.5 years period indicated in the first test (Sec. II).

In that period, 4 Cesium clocks HP5071 were available at IEN, one of them was the master clock generating UTC(IEN). The data at disposal are reported in the Fig. 5. It can be seen that IEN-Cs_2 suffered some anomaly in the last period of observation.

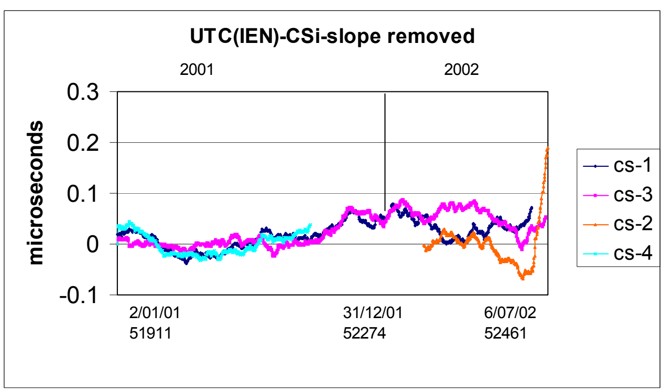

Figure 5. Measured values of UTC(IEN)-Cs_i for the four IEN Cesium clocks

On these series of measures the Allan variance was estimated (Fig.6). The dominant noise is a white frequency noise, resulting in a Wiener process (random walk) on the phase for observation period of 0.5 to about 50 days. It has to be noted this is better than the specified Cesium stability that should reach a flicker floor of $2*10^{-14}$ around a few days.

Using decoupling technique, as the N cornered hat method, it is possible to calculate the Allan variance of each individual clock. That process gave the information that the Cs4 is more noisy than other clocks and on such Cs we will concentrate the further analysis.

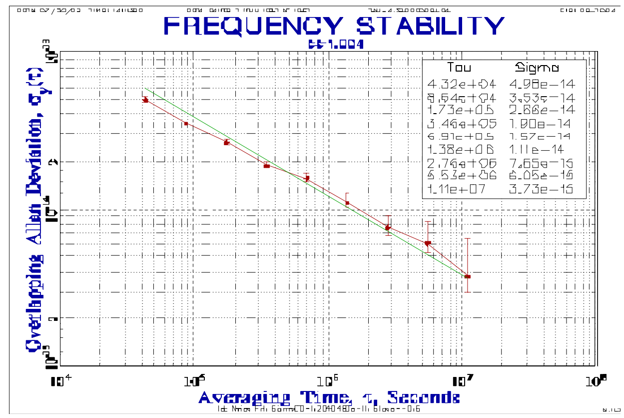

Figure 6. Example of ADEV stability plot obtained with the analysis of IEN cesium data.

The Wiener process is well studied and have at disposal the mathematics to model this behavior, estimating the deterministic frequency offset, and also evaluating a prediction. The Wiener process is characterized by two parameters: diffusion coefficient σ_1 and drift μ [1,2]. The estimation of the values of these two parameters is necessary for the prediction aims. The diffusion coefficient σ_1 is related to the Allan variance, therefore is can be directly estimated by the stability analysis results . The "drift" μ is here intended as linear slope on the clock time error and it is given by the frequency offset. This issue is different from the case of the H maser in which not only a frequency offset is present but also a linear frequency variation (drift) that gives a quadratic variation on the time error. The following analysis is therefore adapt to the Cesium case (Wiener process with linear slope on time error), while for the application to the H-Maser case; the quadratic behavior on the time error should be taken appropriately into account.

Having identified the Wiener process with drift (linear slope plus random walk on the time data), we go back to the IEN Cesium measures referred to the international UTC by means of BIPM Circular T data.

From the theory of Wiener process we know that the linear slope (μ) on time error data can be optimally estimated by evaluating the difference between the last and first time error data, divided by the elapsed time interval, given by the final MJD minus the initial MJD [1]. By calling $x(t)$ the time deviation of a clock, we have

$$\hat{\mu} = \frac{x(MJD_f) - x(MJD_i)}{MJD_f - MJD_i} \quad (1)$$

The uncertainty on such estimate is given by

$$\sigma_{\hat{\mu}}^2 = \frac{\sigma_1^2}{T}$$

where σ_1 is the diffusion coefficient and $T = MJD_f - MJD_i$ the time interval (number of days) between the initial data $x(MJD_i)$ and final data $x(MJD_f)$. From the stability analysis we estimated the Allan deviation $\sigma_y(\tau)$. It is worth recalling the relation between the Allan variance and the diffusion coefficient [3] in case of white frequency noise:

$$\sigma_y^2(\tau) = \frac{\sigma_1^2}{\tau}$$

We take in consideration as an example the IEN Cs_4 that has a larger noise. For $\tau = (8.64*10^4)s = 1$ day, the estimated Allan deviation is $\sigma_y = 3.09*10^{-14}$, giving the estimate $\sigma_1(\tau) = 3.09*10^{-14}$ with time expressed in days. If we evaluate the linear slope μ with $T = 90$ days of measurement, we would have an uncertainty given by

$$\sigma_{\hat{\mu}} = \frac{3.09*10^{-14}}{\sqrt{90}} \cong 3*10^{-15} \cong 0.3\frac{ns}{days}.$$

The uncertainty of the estimate of the linear slope μ is therefore 0.3 nanoseconds/days. If we used the estimate (1) of the linear slope (frequency offset) for the cesium for predicting its behavior, after 45 days of prediction this uncertainty would give an uncertainty of 13.5 nanoseconds (1 σ) and 27 nanoseconds (2 σ) on the prediction only due to the uncertainty in the estimate of the linear slope.

For the evaluation of the complete uncertainty on the prediction we should add also the contribution of the random walk behavior that for a prediction period *t*, would have a variance given by:

$$\sigma_{RW}^2 = \sigma_1^2 * t.$$

In the example considered of the IEN-Cs_4, for t=45 days, we obtain $\sigma_{RW} = 20ns$. By quadratically summing the uncertainty we expected, therefore

$$\sigma_{tot}^2 = (13.5)^2 + (20)^2 = (24)^2, \qquad \sigma_{tot} = 24ns$$

a total uncertainly of 24 ns (1 σ) on the 45 days prediction of the time error of a Cesium clock affected by Random Walk with linear slope.

Let's check from the experimental data if this estimation is confirmed.

We have used the data from UTC-Cs_4. We estimate the linear slope by taking the past BIPM and using (1) above. As much data were available, the longer was the considered observation period T. As longer is the observation interval T, as better should be the linear slope estimation.

In the Fig. 7 the differences between the predicted and real values of (UTC-Cs_4) is shown, this is the prediction errors for the whole test period. We can see that the prediction error grows with time as expected. The standard deviation of the prediction error after 45 days is 27.3 ns not so much different from the expected 24 ns based on the consideration above on the theory of the Wiener process.

Therefore we are quite satisfacted by the Wiener process model. Apart from refinement on the clock model [4] and with the use of a larger set of data that are still in progress, we can now deduce that with one single commercial Cesium clock the UTC prediction for 45 days has a standard deviation in the region of about 30 ns and therefore a prediction error with 95% confidence level would be around 60 ns.

An ensemble of Cs clocks would have improved stability, for example a weighted average of 4 Cesium clocks would have a standard deviation reduced by a factor 2 and it may be expected that, assuming the time error is mostly driven by a Wiener process, i.e. white frequency noise, also the prediction error would be reduced by a factor 2 leading to a region of: UTC prediction error = ± 30 ns with 95% confidence level.

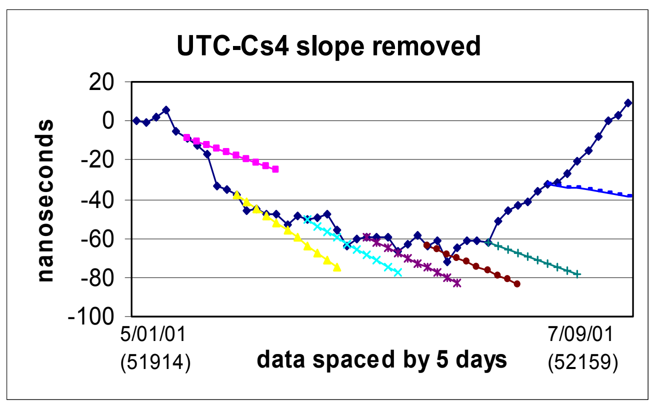

Figure 7. Measured and predicted values of UTC-Cs_{4i} for the 1.5 year test period

V. Conclusion

A first evaluation of the capability of predicting UTC by means of a Cesium clock or a H maser were evaluated. Two different prediction methods were used: the first is based on a simple linear extrapolation of past data, the second is based on the identification of the Cesium stochastic model and on its best prediction. In both cases we realize that with one single commercial Cesium clock, a UTC prediction error in the region of 60 ns (2σ) is the typical achievable situation. A better performance may be achieved by an ensemble of Cesium or by the combined use of Cesium clocks and H-masers. On this way, further studies are in progress.

VI. ACKNOWLEDGMENT

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